EFFECT OF INITIAL FAILURE RATE OF THE FORMATION OF A SPALLING PULSE

A. V. Utkin

UDC 539.593

Studies of spalling phenomena with reflection of a shock wave from the free surface of a body [1, 2] give unique information about the strength properties of materials in the submicrosecond range. However, under these conditions the failure time is comparable with the duration of load operation and failure resistance should be talked about as a function of the deformation rate and other parameters of state. Therefore, it is necessary to obtain data about failure kinetics directly from analysis of experimental data. In implicit form this information contains profiles of test specimen surface movement velocity [3].

In this work wave processes are analyzed in failed material with reflection of a compression pulse from a free surface. The aim of the work is to study the possibilities of obtaining data about the failure rate directly from the results of measuring specimen surface velocity profiles.

Statement and Solution of the Problem. We consider in an acoustic approximation evolution of a triangular compression pulse after reflection of it from a free specimen surface failing with negative pressure. We assume that failure commences on reaching tensile stresses of critical value P_c and it is characterized by specific pore volume v_p. The total specific volume of the material is the sum of v_p and the specific volume of solid component v_s: $v = v_p + v_s$. We use the simplest failure kinetics: the rate of change in v_p is a power function of v_p. Since the initial stage of failure is considered then the rule for collapse of pores with a positive pressure is immaterial. The set of equations of hydrodynamics, closed by equations of kinetics and state, in the Lagrangian variables has the form

$$\frac{\partial v}{\partial t} - \frac{1}{\rho}\frac{\partial u}{\partial h} = 0, \quad \frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial h} = 0, \qquad (1)$$
$$\frac{\partial v_p}{\partial t} + \frac{(\rho v_p)^\alpha}{\rho\tau_\mu} = 0, \quad P = \rho^2 c^2 (1/\rho - v + v_p),$$

where t is time; h is Lagrangian coordinate; u is mass velocity; ρ and c are initial density and speed of sound; and τ_μ is characteristic time for relaxation of the failure process; $\alpha < 1$ is a const. In the equation of state pressure is determined by $v_s = v - v_p$.

Given in Fig. 1 is a picture of flow in plane t–h. In region 1 there is no reaction of incident wave with the reflected wave and the dependence of mass velocity on pressure coordinate and time corresponds to the triangular compression pulse:

$$u(h,\ t) = u_0 - k(ct - h), \quad P(h,\ t) = \rho c u(h,\ t). \qquad (2)$$

Here u_0 is maximum mass velocity; k is a constant characterizing pulse duration $2h_0$:

$$h_0 = -c\tau_0 = -u_0/2k.$$

In region 3 there is reaction of the incident pulse and that reflected from the free surface h = 0 which leads to occurrence of tensile stresses. Their absolute value does not exceed the critical value; therefore, the material does not fail and the solution satisfying the condition at the free surface has the form

$$u(h,\ t) = 2(u_0 - kct), \quad P(h,\ t) = 2\rho ckh. \qquad (3)$$

Chernogolovka. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 4, pp. 140-146, July-August, 1993. Original article submitted June 29, 1992.

 0021-8944/93/3404-0578$12.50

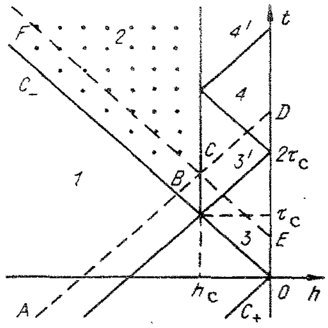

Fig. 1

With $h = h_c$, $t = \tau_c = -h_c/c$ the pressure reaches a threshold P_c and in region 2 there is material failure. Here flow is determined as a result of solving set (1) with boundary conditions with $h = h_c$ and $h \to -\infty$ and initial conditions for the C-characteristic at which the functions in question, with the exception of v_p, undergo a jump. It should be noted that due to stress relaxation with failure pressure along the C-characteristic may appear to be (with certain values of τ_μ) higher than P_c and the failure region will have a more complex zonal structure compared with that given in Fig. 1. This case corresponding to multiple spalling is analyzed in detail below. Now we assume that after reaching the failure threshold in section h_c with lower values of h the material 'weakens.'

We find a solution in region 2. For this we exclude from (1) v_p and v and substitute independent variables:

$$T = t + h/c, \quad x = h.$$

The failure zone is depicted in part of the fourth quadrant of plane T – x: $T \geq 0$, $x \leq x_c$. The set of two equations obtained in partial derivatives after applying Laplace transformations to it with respect to T is converted into a set of normal differential equations

$$\frac{d\widehat{u}}{dx} + \frac{s}{c}\widehat{u} + s\frac{\widehat{P}}{\rho c^2} = \frac{1}{\rho c^2}(P(x,0) + \rho c u(x,0)) + F(s), \qquad (4)$$
$$\frac{d\widehat{P}}{dx} + \frac{s}{c}\widehat{P} + \rho s\widehat{u} = \frac{1}{c}(P(x,0) + \rho c u(x,0)),$$

where s is Laplace variable; $\hat{u}$ and $\hat{P}$ are Laplace images of mass velocity and pressure; F(s) is a Laplace image for failure rate ρv_p whose dependence on time has the form (the dot signifies a partial derivative with respect to t or with respect to T since they coincide)

$$\rho\dot{v}_p = \frac{1}{\tau_\mu}\left((1-\alpha)\frac{T}{\tau_\mu}\right)^{\alpha/(1-\alpha)}. \qquad (5)$$

In the right-hand part of (4) initial values of u and P with $T \to +0$ are transposed entering in the form of a combination which has a Reimann J_+-invariant [4]. Therefore it is not necessary to determine u and P separately to the right of the jump in the C-characteristic: they will be found directly from solving the set. The value of the invariant is found from the continuity condition at the jump from its value in region 1. In accordance with (2) we obtain

$$P(x,0) + \rho c u(x,0) = 2\rho c(u_0 + 2kx)\theta(x - x_0)$$

($\theta(x)$ is the Heavyside unit function, $x_0 = h_0$).

The general solution in the failure region not growing exponentially with $x \to -\infty$ is written in the form

$$\widehat{P}(x,s) = \frac{2k\rho c}{s}\left[x - x_0 - \frac{c}{2s}\left(1 - \exp\frac{-2s(x-x_0)}{c}\right)\right]\theta(x - x_0) -$$
$$-\frac{\rho c}{2}F(s)\left(x - \frac{c}{2s}\right) + b,$$

$$\widehat{u}(x, s)=\frac{2k}{s}\left[x-x_0-\frac{c}{2s}\left(1-\exp\frac{-2s(x-x_0)}{c}\right)\right]\theta(x-x_0)+\frac{\rho c}{2}F(s)\left(x+\frac{c}{2s}\right)-b \tag{6}$$

(the linear increase in $\hat{P}$ with $x \to -\infty$ is caused by exclusion from the consideration of pore collapse kinetics with positive pressure). Constant b is found from the continuity condition for the Reimann J_--invariant with $x = x_c$. In regions 3', 4', etc. (see Fig. 1) the functional dependence of J_- on coordinate and time is different and the invariant in each subsequent region is only determined after finding the solution in the previous region. We find the value of constant b in the range $0 \leq T \leq 2\tau_c$. In region 3 in accordance with (3) we have

$$J_-=-2\rho c[u_0-k(ct+h)]=-2\rho c(u_0-kcT). \tag{7}$$

Since the J_--invariant is retained along the C_--characteristic, then relationship (7) gives its value in region 3'. By applying a Laplace transformation to (7) and equating the expression obtained to the $\hat{J}_-$-invariant in the failure region following from (6) with $x = x_c$ we find that

$$b=\frac{k\rho c}{s}\left(2x_0+\frac{c}{s}\right)+\frac{\rho c}{2}F(s)x_c. \tag{8}$$

Equations (6) and (8) give a solution in the failure region with $0 < T \leq 2\tau_c$ in Laplace images. Some results may be obtained directly from (6) without conversion to originals. For example, by using the known property of Laplace transformation [5] $\lim_{s\to\infty} sG(s) = G(0)$ we find the value of pressure to the right of the jump along the C_--characteristic with $h > h_0$:

$$P=2\rho ckh-\frac{\rho c}{2}(h-h_c)L \tag{9}$$

($L = 0$ with $0 < \alpha < 1$ and $L = 1/\tau_\mu$ with $\alpha = 0$), i.e., if the initial failure rate equals zero, then pressure immediately behind the jump changes the same as in the absence of failure, and in particular with $h \leq h_c$ after reaching P_c it continues to decrease. Therefore, the failure region in this case is not limited with $h \to -\infty$ and multiple spalling is impossible. With $\alpha = 0$ the situation is different: after the start of failure at point h_c, τ_c pressure along the C_--characteristic continues to decrease if $\tau_\mu > 1/4k$, and it starts to grow if $\tau_\mu < 1/4k$. With $\tau_\mu = 1/4k$ pressure remains constant and equal to P_c. Thus, the assumption made about material 'weakening' is significant with low fracture toughness when $\tau_\mu > 1/4k$.

We find the free surface velocity with $2\tau_c \leq t \leq 4\tau_c$. For this we use the situation that the J_+-invariant is maintained along the C_+-characteristic. Its value at the free surface equals $\rho c u(0, t)$, and with $h = h_c$ from the solution obtained in the failure region we find that

$$\frac{\widehat{J}_+(x_c, s)}{\rho c}=\frac{4k}{s}(x_c-x_0)-\frac{2kc}{s^2}\left(1-\exp\frac{-2s(x_c-x_0)}{c}\right)+\frac{c}{2s}F(s).$$

By using known inversion formulas and the properties of Laplace transformation [5, 6], for the free surface velocity with $2\tau_c \leq t \leq 4\tau_c$ we have

$$\frac{u(0,t)}{2u_0}=1-\frac{t}{2\tau_0}+\frac{c}{4u_0}\left[(1-\alpha)\frac{t-2\tau_c}{\tau_\mu}\right]^{1/(1-\alpha)}. \tag{10}$$

Analysis of the Solution.

1. $\alpha = 0$, $F(s) = 1/(s\tau_\mu)$. This case corresponds to a constant failure rate and the solution obtained has the simplest form.

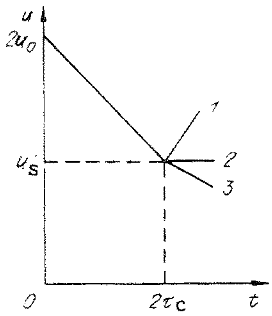

Fig. 2

Velocity profiles with different τ_μ plotted by Eq. (10) are given in Fig. 2. It can be seen that as also for pressure there exists a critical value of characteristic failure time equal to 1/4k (line 2 in Fig. 1) with which the free surface velocity after the commencement of failure of remains constant: $u(0,\ t) = u_S = 2u_0 - 4kc\tau_C$. With $\tau_\mu < 1/4k$ (line 1) failure develops in the form of a spalling pulse in the profile u(0, t), and with $\tau_\mu > 1/4k$ (line 3) after the start of failure the fall in velocity continues. It is convenient to present the result obtained in the following formulation by introducing the failure rate $\dot{v}_p = 1/\rho\tau_\mu$ and the expansion rate for a specific volume $\dot{v} = k/\rho$ in the unloaded part of the incident pulse: a spalling pulse in the free surface velocity profile only forms in the case when the initial failure rate exceeds by more than a factor of four the failure rate for a specific volume in the unloaded part of the incident pulse; the curvature of the spalling pulse front is clearly determined by the failure rate:

$$\frac{d}{dt}\left(\frac{u(0,t)}{2u_0}\right) = \frac{1}{8\tau_0}\left(\frac{\dot{v}_p}{\dot{v}} - 4\right) \text{with } t > 2\tau_C.$$

We also consider the change in state of a substance along the characteristic on coordinates P—u. The solution in the failure region emerging from (6) after using inverse transformation has the form

$$P(h,\ t) = 2\rho ckh + \rho c^2(t - h/c - 2\tau_C)/4\tau_\mu,$$
$$u(h,\ t) = 2(u_0 - kct) + c(t + 3h/c + 2\tau_C)/4\tau_\mu.$$

whence we obtain the connection between P and u along the C_+-characteristic in section BC (see Fig. 1):

$$P - P_+ = \frac{\rho c}{1/(2k\tau_\mu) - 1}(u - u_+) \tag{11}$$

[P_+ and u_+ are values of pressure and mass velocity at the point of intersection of the C_+-characteristic with straight line $h = h_C$ (at point C in Fig. 1)]. In regions 3' and 4 along this characteristic

$$P - P_+ = -\rho c(u - u_+). \tag{12}$$

It can be seen from (11) that trajectories for the change in state along the characteristic on coordinates P, u deviate from the straight line determined by Reimann invariants in the direction of an increase in mass velocity: the failure process affects not only the value, but also the sign of the slope of the trajectory whose change occurs with failure rates half ($\tau_\mu = 1/2k$) those which are necessary for occurrence of a minimum on the free surface velocity profile.

Shown in Fig. 3 is the nature of change in state of particles along the C_+-characteristic ABCD (see Fig. 1) with $\tau_\mu = 1/4k$. Arrows indicate the direction of movement. The initial states lie on straight line ON. After encountering the leading characteristic of the rarefaction wave the state of the jump changes from A into B and in the failure region

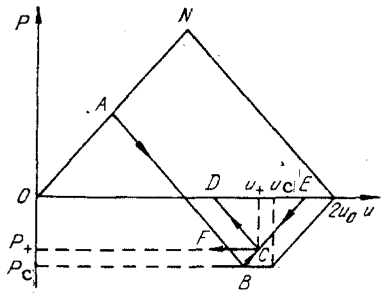

Fig. 3

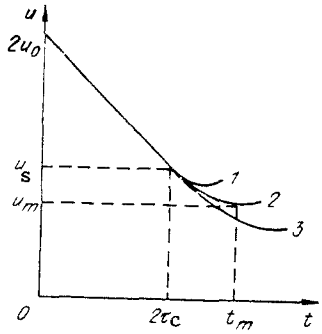

Fig. 4

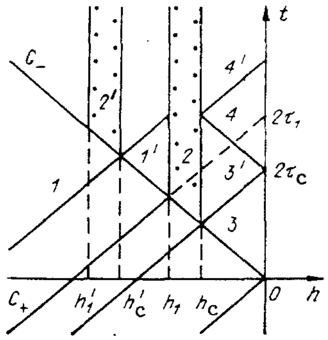

Fig. 5

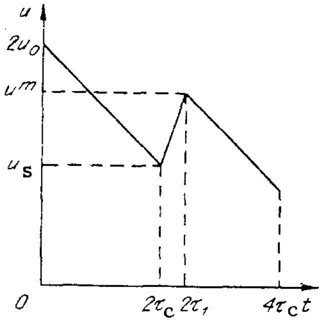

Fig. 6

it changes along BC. In the unfailed part of a specimen in section CD the connection between P and u is given by relationship (12). With a prescribed value of τ_μ when a minimum first appears in the free surface velocity profile (i.e., $u(0, t) = \text{const}$ with $t > 2\tau_c$) there is merging of trajectories for the change in states in plane P–u in section CD along all C_+-characteristics intersecting in the failure region, i.e., relationship (12) is common for them. This fact was noted in [7].

Also shown in Figs. 1 and 3 are C_--characteristic ECF and the trajectory for a change in state corresponding to it. In the failure region in section CF pressure remains constant. In the general case for the C_--characteristic the connection between P and u has the form

$$C_-\colon\ P - P_- = -\rho c \frac{1 - 4k\tau_\mu}{1 + 4k\tau_\mu}(u - u_-),$$

where P_- and u_- are pressure and mass velocity at the point of intersection of the C_--characteristic with $h = h_c$. For this the case $P_- = P_+$ and $u_- = u_+$ is given in the diagrams.

We note that the solution found is correct on condition that pressure is negative in that part of the failure region which affects the free surface movement velocity.

2. $\alpha > 1$. In this case the initial failure rate is zero. Given in Fig. 4 is the dependence of free surface velocity on time plotted by Eq. (10). Curves 1-3 relate to an increase in either α or τ_μ. In contrast to the constant failure rate with $t = 2\tau_c$ the derivative of free surface velocity is continuous and a minimum is achieved for $t = t_m > 2\tau_c$:

$$t_m = 2\tau_c + \frac{\tau_\mu}{1-\alpha}(4k\tau_\mu)^{(1-\alpha)/\alpha}. \tag{13}$$

It is easy to find the corresponding value of velocity u_m by substituting (13) in (10). In practice spalling resistance σ^* is normally determined by the difference in velocities $\Delta w = 2u_0 - u_m$: $\sigma^* = 0.5\rho c\Delta w$. With constant failure rate $\sigma^* = -P_c$. For the case in question this equality is not fulfilled and spalling resistance depends both on failure kinetics and on the rate of expansion of a specific volume in the unloaded part of the incident pulse:

$$\sigma^* = -P_c + \frac{\rho c^2 \alpha}{4(1-\alpha)}(4\tau_\mu \rho \dot{v})^{1/\alpha}.$$

Whence it follows in particular that presentation of experimental data on coordinates σ^*—$\dot{v}$ [3] carries useful information about failure kinetics. Tensile stresses first reach $-\sigma^*$ in section $h^* = -\sigma^*/(2k\rho c) < h_c$, i.e., the thickness of spalled plate determined from the minimum in the free surface velocity profile in this case exceeds the true value of h_c.

We find the failure rate with $h = h_c$ at instant of time t relating to t_m: $t = t_m - \tau_c$. By substituting this value of t in (5) we obtain $\dot{v}_p = 4k/\rho = 4\dot{v}$, i.e., as with a constant failure rate the minimum on the free surface velocity profile, and consequently, the start of spalling pulse formation, is observed when the failure rate in the spalling plane equals the quadrupled failure rate for a specific volume in the unloaded part of the incident pulse.

Multiple Spalling. Now we consider specimen failure with $\alpha = 0$ without assuming that the failure region is limited with $h \to -\infty$. Let after the start of failure at point h_c the threshold for the start of failure with $h < h_c$ decrease to εP_c ($\varepsilon < 1$). Then with $\tau_\mu < 1/4k$ the structure of flow in plane t—h has the form presented in Fig. 5. The left-hand boundary of failure region 2 ($h = h_1$) is determined from the condition for equality of pressure εP_c.

As before, the general solution in Laplace images is determined by solving set (4). Two constants in this case are found from the continuity condition for the J_+-invariant with $h = h_1$ and the J_--invariant with $h = h_c$. Without dwelling on intermediate calculations we give some final results. As before the change in pressure of a jump along the C_--characteristic with $h_1 < h < h_c$ is described by relationship (9) from which in particular it follows that $h_1 = h_c(1 - 4k\tau_\mu\varepsilon)/(1 - 4k\tau_\mu)$. With $\tau_\mu \to 0$ the size of the failure region tends toward zero ($h_1 \to h_c$) as it should be with instantaneous spalling. In the second limiting case ($\tau_\mu \to 1/4k$) the failure region increases without restriction, i.e., we change over to the solution considered above. Free surface movement velocity has the form

$$\frac{u(0,t)}{2u_0} = 1 - \frac{t}{2\tau_0} + \frac{c}{4u_0\tau_\mu}(t - 2\tau_c) - \frac{c}{4u_0\tau_\mu}(t - 2\tau_1)\,\theta(t - 2\tau_1), \tag{14}$$

where $\tau_1 = -h_1/c$.

Given in Fig. 6 is a typical free surface velocity profile plotted by Eq. (14). With $t < 2\tau_1$ the solution of (14) agrees with (10), i.e., consideration of the situation that the failure zone is finite does not affect the formation of the spalling pulse front. With $t > 2\tau_1$ the rate starts to decrease and with the same gradient as in the original incident pulse in the range $0 < t < 2\tau_c$ (fulfillment of the inequality $\tau_1 < 2\tau_c$ is assumed, which is correct with $4k\tau_\mu < 1/(2 - \varepsilon)$). At point $2\tau_1$ the spalling pulse reaches a maximum ($u^m/(2u_0) = 1 - \varepsilon\tau_c/\tau_0$) which does not depend on failure rate. This is a drawback of the model in question in which in particular no failure criterion is introduced. The second minimum on the velocity profile referred to $2u_0$ and corresponding to time $4\tau_c$ equals $1 - (\tau_c/\tau_0)(1 + (\varepsilon - 4k\tau_\mu)/(1 - 4k\tau_\mu))$, i.e., it lies below or above the first minimum u_c depending on the sign of $\varepsilon - 4k\tau_\mu$.

In a similar way it is possible to obtain a solution with $h < h_1$ (Fig. 5). In the range $h_c' < h < h_1$ a specimen does not fail and the solution is found by the method of characteristics. From it, in particular, it follows that pressure behind the jump to the right of the C_--characteristic increases in absolute value with a reduction in h by the rule

$$P = 2k\rho ch + \frac{\rho c}{2\tau_\mu}(h_c - h_1),$$

reaching the threshold for the start of failure P_c at point h_c', and the thickness of the unfailed part of the specimen $h_1 - h_c' = -(1 - \varepsilon)h_c$. The second failure zone occupies the region $h_1' < h < h_c'$. The dependence of pressure on h immediately behind the jump is given in this case by the relationship

$$P = 2k\rho ch + \frac{\rho c}{2\tau_\mu}(h_c - h_1 + h_c' - h),$$

from which it follows that the sizes of the first and second failure zones coincide: $h_c' - h_1' = h_c - h_1$.

The results provided show that the finiteness of the failure zone markedly affects the form of the spalling pulse as a whole without reflecting on front formation. We also note that as mentioned above with a zero initial failure rate ($\alpha > 0$) the failure region is always infinite and multiple spalling is generally impossible if no failure criterion is introduced.

Thus, with the scope of acoustics an analytical expression is obtained for free surface velocity with constant and zero initial material failure rates. A critical value of failure rate is found with which formation of a spalling pulse commences. A method is suggested for determining the initial rate of an increase in specific pore volume over the curvature of the spalling pulse front not connected with a specific failure model. It is shown that spalling resistance, determined from the drop in free surface movement velocity, depends in the general case on the failure rate for a specific volume in the unloaded part of the incident pulse.

LITERATURE CITED

1. S. A. Novikov, I. I. Divnov, and A. G. Ivanov, "Study of the failure of steel, aluminum, and copper with explosive loading," Fiz. Met. Metalloved., 25, No. 4 (1964).
2. G. I. Kanel' and S. V. Razorenov, "Shock-wave loading of metals. Specimen surface movement," Preprint, OIKhF Akad. Nauk SSSR, Chernogolovka (1989).
3. G. I. Kanel' and V. E. Fortov, "Mechanical properties of condensed material with intense pulse effects," Usp. Mekh., 19, No. 3 (1987).
4. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow (1973).
5. M. A. Lavrent'ev and B. V. Shabat, Methods of Complex Variable Function Theory [in Russian], Nauka, Moscow (1973).
6. M. Abramovits and I. Stigan (eds.), Handbook for Special Functions [Russian translation], Nauka, Moscow (1979).
7. G. I. Kanel' and L. G. Chernykh, "The process of spalling failure," Prikl. Mekh. Tekh. Fiz., No. 6 (1980).